## Outline

Chapter 1 Background and Fundamentals of Mathematics
Sets, Cartesian products ..... 1
Relations, partial orderings, Hausdorff maximality principle, ..... 3
equivalence relations
Functions, bijections, strips, solutions of equations, ..... 5
right and left inverses, projections
Notation for the logic of mathematics ..... 13
Integers, subgroups, unique factorization ..... 14
Chapter 2 Groups
Groups, scalar multiplication for additive groups ..... 19
Subgroups, order, cosets ..... 21
Normal subgroups, quotient groups, the integers mod $n$ ..... 25
Homomorphisms ..... 27
Permutations, the symmetric groups ..... 31
Product of groups ..... 34
Chapter 3 Rings
Rings ..... 37
Units, domains, fields ..... 38
The integers mod $n$ ..... 40
Ideals and quotient rings ..... 41
Homomorphisms ..... 42
Polynomial rings ..... 45
Product of rings ..... 49
The Chinese remainder theorem ..... 50
Characteristic ..... 50
Boolean rings ..... 51
Chapter 4 Matrices and Matrix Rings
Addition and multiplication of matrices, invertible matrices ..... 53
Transpose ..... 55
Triangular, diagonal, and scalar matrices ..... 56
Elementary operations and elementary matrices ..... 57
Systems of equations ..... 59
Determinants, the classical adjoint ..... 60
Similarity, trace, and characteristic polynomial ..... 64
Chapter 5 Linear Algebra
Modules, submodules ..... 68
Homomorphisms ..... 69
Homomorphisms on $R^{n}$ ..... 71
Cosets and quotient modules ..... 74
Products and coproducts ..... 75
Summands ..... 77
Independence, generating sets, and free basis ..... 78
Characterization of free modules ..... 79
Uniqueness of dimension ..... 82
Change of basis ..... 83
Vector spaces, square matrices over fields, rank of a matrix ..... 85
Geometric interpretation of determinant ..... 90
Linear functions approximate differentiable functions locally ..... 91
The transpose principle ..... 92
Nilpotent homomorphisms ..... 93
Eigenvalues, characteristic roots ..... 94
Jordan canonical form ..... 96
Inner product spaces, Gram-Schmidt orthonormalization ..... 98
Orthogonal matrices, the orthogonal group ..... 102
Diagonalization of symmetric matrices ..... 103
Chapter 6 Appendix
The Chinese remainder theorem ..... 108
Prime and maximal ideals and UFD ${ }^{s}$ ..... 109
Splitting short exact sequences ..... 114
Euclidean domains ..... 116
Jordan blocks ..... 122
Jordan canonical form ..... 123
Determinants ..... 128
Dual spaces ..... 130

