Outline

Chapter 1 Background and Fundamentals of Mathematics

Sets, Cartesian products	1
Relations, partial orderings, Hausdorff maximality principle,	3
equivalence relations	
Functions, bijections, strips, solutions of equations,	5
right and left inverses, projections	
Notation for the logic of mathematics	13
Integers, subgroups, unique factorization	14

Chapter 2 Groups

19
21
25
27
31
34

Chapter 3 Rings

Rings	37
Units, domains, fields	38
The integers mod n	40
Ideals and quotient rings	41
Homomorphisms	42
Polynomial rings	45
Product of rings	49
The Chinese remainder theorem	50
Characteristic	50
Boolean rings	51

Chapter 4 Matrices and Matrix Rings

Addition and multiplication of matrices, invertible matrices	53
Transpose	55
Triangular, diagonal, and scalar matrices	56
Elementary operations and elementary matrices	57
Systems of equations	59

Determinants, the classical adjoint	60
Similarity, trace, and characteristic polynomial	64

Chapter 5 Linear Algebra

Modules, submodules	68
Homomorphisms	69
Homomorphisms on \mathbb{R}^n	71
Cosets and quotient modules	74
Products and coproducts	75
Summands	77
Independence, generating sets, and free basis	78
Characterization of free modules	79
Uniqueness of dimension	82
Change of basis	83
Vector spaces, square matrices over fields, rank of a matrix	85
Geometric interpretation of determinant	90
Linear functions approximate differentiable functions locally	91
The transpose principle	92
Nilpotent homomorphisms	93
Eigenvalues, characteristic roots	94
Jordan canonical form	96
Inner product spaces, Gram-Schmidt orthonormalization	98
Orthogonal matrices, the orthogonal group	102
Diagonalization of symmetric matrices	103

Chapter 6 Appendix

The Chinese remainder theorem	108
Prime and maximal ideals and UFD^s	109
Splitting short exact sequences	114
Euclidean domains	116
Jordan blocks	122
Jordan canonical form	123
Determinants	128
Dual spaces	130

vii